In the Name of God

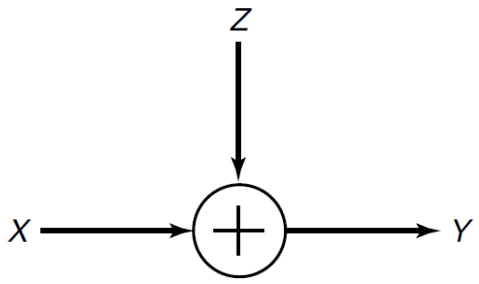
Digital Communications

Assignment #2

Discrete Channel Capacity

**1.**

*Additive noise channel.* Find the channel capacity of the following discrete memoryless channel:




where $\Pr\{Z = 0\} = \Pr\{Z = a\} = \frac{1}{2}$. The alphabet for $x$ is $\mathbf{X} = \{0, 1\}$. Assume that $Z$ is independent of $X$. Observe that the channel capacity depends on the value of $a$.

**2.**

*Channel capacity.* Consider the discrete memoryless channel $Y = X + Z \pmod{11}$, where

$$Z = \begin{pmatrix} 1, & 2, & 3 \\ \frac{1}{3}, & \frac{1}{3}, & \frac{1}{3} \end{pmatrix}$$

and $X \in \{0, 1, \ldots, 10\}$. Assume that $Z$ is independent of $X$.

**(a)** Find the capacity.

**(b)** What is the maximizing $p^*(x)$?

3.

*Z-channel.* The $Z$-channel has binary input and output alphabets and transition probabilities $p(y|x)$ given by the following matrix:

$$Q = \begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix} \qquad x, y \in \{0, 1\}$$

Find the capacity of the $Z$-channel and the maximizing input probability distribution.