Assignment Number 7

Stochastic Processes course,

Semester 1, 90-91

Science and Research Branch in Azad University

1. Consider a random process, $X(t)$ defined by,

$$X(t) = Y \cos wt, \quad t \geq 0$$

Where w is a constant and Y is a uniform r. v. over (0, 1)

a) Find $E[X(t)]$
b) Find the autocorrelation function $R_x(t, s)$ of $X(t)$
c) Find the autocovariance function $C_x(t, s)$ of $X(t)$

2. Consider a discrete-parameter random process $X(n) = \{X_n, n \geq 1\}$ where the X_n 's are iid r. v.'s with common cdf $F_x(x)$, mean μ, and variance σ^2

a) Find the joint cdf of $X(n)$
b) Find $E[X(n)]$
c) Find the autocorrelation function $R_x(n, m)$ of $X(n)$
d) Find the autocovariance function $C_x(n, m)$ of $X(n)$

3. Consider a random process, $X(t)$ defined by,

$$X(t) = U \cos t + V \sin t, \qquad -\infty < t < \infty$$

Where U and V are independent r. v.'s, each of which assumes the values -2 and 1 with the probabilities 1/3 and 2/3, respectively. Show that $X(t)$ is WSS but not SSS.

4. Consider a random process, $X(t)$ defined by,

$$X(t) = A \cos(wt+\Theta), \qquad -\infty < t < \infty$$

Where A and w are constant and Θ is a uniform r. v. over $(-\pi, \pi)$. Show that $X(t)$ is WSS.

5. Let $\{X(t), -\infty < t < \infty\}$ be a zero-mean, stationary, normal process with the autocorrelation function

$$R_X(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T}, & -T \le \tau \le T \\ 0, & Otherwise \end{cases}$$

Let { $X(t_i)$, $i = 1, 2, \ldots, n$ } be a sequence of n samples of the process taken at the time instants $t_i = iT/2$, $i = 1, 2, \ldots, n$. find the mean and the variance of the sample mean $\hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} X(t_i)$.

6. Let $\{X_n, n \ge 0\}$ be a sequence of iid r. v.'s with mean 0 and variance 1. Show $\{X_n, n \ge 1\}$ is a WSS process.

7. Suppose t is not a point at which an event occurs in a Poisson process $X(t)$ with rate λ. Let $W(t)$ be the r. v. representing the time until the next occurrence of an event. Show that the distribution of $W(t)$ is independent of t and $W(t)$ is an exponential r. v. with parameter λ.

8. Using the notion of generalized derivative, show that the generalized derivative $X'(t)$ of the Wiener process $X(t)$ is a white noise.

9. Let $X(t)$ is a Poisson process with rate λ and $Y(t) = X(t) - \lambda t$. Show that the generalized derivative $Y'(t)$ of the $Y(t)$ is a white noise.

10. Let $X(t)$ is a Poisson process with rate λ. Find the mean and correlation functions of the generalized derivative $X'(t)$ of the Poisson process $X(t)$. (This process is also called as Poisson's impulses)