**1.**

*Huffman coding.* Consider the random variable

$$X = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ 0.49 & 0.26 & 0.12 & 0.04 & 0.04 & 0.03 & 0.02 \end{pmatrix}.$$

**(a)** Find a binary Huffman code for $X$.

**(b)** Find the expected code length for this encoding.

**(c)** Find a ternary Huffman code for $X$.

**2.**

*More Huffman codes.* Find the binary Huffman code for the source with probabilities $(\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{2}{15}, \frac{2}{15})$. Argue that this code is also optimal for the source with probabilities $(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5})$.

**3.**

*Bad codes.* Which of these codes cannot be Huffman codes for any probability assignment?

**(a)** $\{0, 10, 11\}$

**(b)** $\{00, 01, 10, 110\}$

**(c)** $\{01, 10\}$

**4.**

*Huffman code.* Find the **(a)** *binary* and **(b)** *ternary* Huffman codes for the random variable $X$ with probabilities

$$p = \left( \frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21} \right).$$

**(c)** Calculate $L = \sum p_i l_i$ in each case.