In the Name of God

Digital Communications

Assignment #1

Concepts of Information Theory

1.

Entropy of functions of a random variable. Let X be a discrete random variable. Show that the entropy of a function of X is less than or equal to the entropy of X by justifying the following steps:

$$\begin{aligned} H(X, g(X)) &\stackrel{(a)}{=} H(X) + H(g(X) \mid X) \\ &\stackrel{(b)}{=} H(X), \\ H(X, g(X)) &\stackrel{(c)}{=} H(g(X)) + H(X \mid g(X)) \\ &\stackrel{(d)}{\geq} H(g(X)). \end{aligned}$$

Thus, $H(g(X)) \leq H(X)$.

2.

Example of joint entropy. Let $p(x, y)$ be given by

$X \backslash Y$	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

Find:

(a) $H(X)$, $H(Y)$.

(b) $H(X \mid Y)$, $H(Y \mid X)$.

(c) $H(X, Y)$.

(d) $H(Y) - H(Y \mid X)$.

(e) $I(X; Y)$.

(f) Draw a Venn diagram for the quantities in parts (a) through (e).

3.

Discrete entropies. Let X and Y be two independent integer-valued random variables. Let X be uniformly distributed over $\{1, 2, \ldots, 8\}$, and let $\Pr\{Y = k\} = 2^{-k}$, $k = 1, 2, 3, \ldots$.

(a) Find $H(X)$.

(b) Find $H(Y)$.

(c) Find $H(X + Y, X - Y)$.